



General Certificate of Education

Mathematics 6360

MS2B Statistics 2B

Mark Scheme

2009 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

Copyright © 2009 AQA and its licensors. All rights reserved.

COPYRIGHT

AQA retains the copyright on all its publications. However, registered centres for AQA are permitted to copy material from this booklet for their own internal use, with the following important exception: AQA cannot give permission to centres to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Set and published by the Assessment and Qualifications Alliance.

Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Q	Solution	Marks	Total	Comments
1	$H_0: \mu = 768$ $H_1: \mu \neq 768$ Test statistic: $z = \frac{764.8 - 768}{\frac{8}{\sqrt{18}}}$ $= -1.70$ $z_{crit} = \pm 1.96$ \Rightarrow Accept H_0 No evidence at the 5% level of significance, to deny Yvonne's claim.	B1 M1 A1 B1 A1 E1	 6	(Both) (-1.697) ($z_{crit} = 1.96$ or $z_{crit} = -1.96$)
	Total		6	
2(a)(i)	$X \sim \text{Po}(5.0)$ $\Rightarrow P(X < 4) = P(X \leq 3)$ $= 0.265$	B2	2	(0.440 to 0.441) for B1 CAO
(ii)	$Y \sim \text{Po}(1.5)$ $\Rightarrow P(Y = 4) = \frac{e^{-1.5} \times (1.5)^4}{4!}$ $= 0.0471$	M1 A1	2	(0.047 to 0.0471)
2(b)(i)	$T = X + Y \sim \text{Po}(6.5)$ $\Rightarrow P(T > 5) = 1 - P(T \leq 5)$ $= 1 - 0.369$ $= 0.631$	B1 B1 B1	3	(1 - 0.2237) or (1 - 0.5265)
(ii)	$p = {}^8C_7 (0.631)^7 (0.369) +$ $(0.631)^8$ $p = 0.11758 + 0.02513$ $= 0.143$	M1ft A1ft A1	3	ft on their p from (b)(i) Either part attempted (both parts correct) AWFW 1.142 to 0.143 (CAO)
(c)(i)	Mean = 8 Variance = $s^2 = 16.9$ (sample variance = 15.2)	B1 B1	2	CAO (AWRT)
(ii)	Poisson not a good model for data Mean \neq Variance	B1dep B1	2	
	Total		14	

MS2B (cont)

Q	Solution	Marks	Total	Comments																																																																																												
3	<p>H_0 : no association between age and attitude to school reorganisation H_1 : association between age and attitude to school reorganisation</p> <table border="1"> <thead> <tr> <th>Age</th> <th colspan="2">Against</th> </tr> <tr> <th></th> <th>O_i</th> <th>E_i</th> </tr> </thead> <tbody> <tr> <td>16 - 17</td> <td>9</td> <td>$6\frac{17}{65}$</td> </tr> <tr> <td>18 - 21</td> <td>17</td> <td>$15\frac{24}{65}$</td> </tr> <tr> <td>22 - 49</td> <td>115</td> <td>$116\frac{9}{13}$</td> </tr> <tr> <td>50 - 65</td> <td>41</td> <td>$42\frac{9}{13}$</td> </tr> <tr> <td>> 65</td> <td>3</td> <td>$3\frac{64}{65}$</td> </tr> <tr> <td>Total</td> <td>185</td> <td>185</td> </tr> </tbody> </table> <table border="1"> <thead> <tr> <th>Age</th> <th colspan="2">Not Against</th> </tr> <tr> <th></th> <th>O_i</th> <th>E_i</th> </tr> </thead> <tbody> <tr> <td>16 - 17</td> <td>2</td> <td>$4\frac{48}{65}$</td> </tr> <tr> <td>18 - 21</td> <td>10</td> <td>$11\frac{41}{65}$</td> </tr> <tr> <td>22 - 49</td> <td>90</td> <td>$88\frac{4}{13}$</td> </tr> <tr> <td>50 - 65</td> <td>34</td> <td>$32\frac{4}{13}$</td> </tr> <tr> <td>> 65</td> <td>4</td> <td>$3\frac{1}{65}$</td> </tr> <tr> <td>Total</td> <td>140</td> <td>140</td> </tr> </tbody> </table> <p>Row totals: $\widehat{11,27}$ 205, $\widehat{75.7}$ (325) Column totals: 185, 140 (325) $E_i's < 5$ \therefore combine cells 16 – 17 and 18 –21 also 50 – 65 and ‘over 65’ to give:</p> <table border="1"> <thead> <tr> <th>O_i</th> <th>E_i</th> <th>$\alpha = O_i - E_i$</th> <th>$\frac{\alpha^2}{E_i}$</th> </tr> </thead> <tbody> <tr> <td>26</td> <td>21.63</td> <td>4.369</td> <td>0.8825</td> </tr> <tr> <td>115</td> <td>116.69</td> <td>-1.692</td> <td>0.0245</td> </tr> <tr> <td>44</td> <td>46.68</td> <td>-2.677</td> <td>0.1535</td> </tr> <tr> <td>12</td> <td>16.37</td> <td>-4.369</td> <td>1.1662</td> </tr> <tr> <td>90</td> <td>88.31</td> <td>1.692</td> <td>0.0324</td> </tr> <tr> <td>38</td> <td>35.32</td> <td>2.677</td> <td>0.2029</td> </tr> <tr> <td>325</td> <td>325</td> <td></td> <td>2.462</td> </tr> </tbody> </table> <p>$X^2 = 2.462$ $\nu = 2$ $\chi^2_{\nu=2} (0.95) = 5.991$ Accept H_0 No real evidence at 5% level of significance to suggest any association between age and attitude to school reorganisation.</p>	Age	Against			O_i	E_i	16 - 17	9	$6\frac{17}{65}$	18 - 21	17	$15\frac{24}{65}$	22 - 49	115	$116\frac{9}{13}$	50 - 65	41	$42\frac{9}{13}$	> 65	3	$3\frac{64}{65}$	Total	185	185	Age	Not Against			O_i	E_i	16 - 17	2	$4\frac{48}{65}$	18 - 21	10	$11\frac{41}{65}$	22 - 49	90	$88\frac{4}{13}$	50 - 65	34	$32\frac{4}{13}$	> 65	4	$3\frac{1}{65}$	Total	140	140	O_i	E_i	$\alpha = O_i - E_i$	$\frac{\alpha^2}{E_i}$	26	21.63	4.369	0.8825	115	116.69	-1.692	0.0245	44	46.68	-2.677	0.1535	12	16.37	-4.369	1.1662	90	88.31	1.692	0.0324	38	35.32	2.677	0.2029	325	325		2.462	<p>B1</p> <p>M1 A1</p> <p>B1</p> <p>M1 A1</p> <p>ml</p> <p>A1 B1 B1ft A1ft</p> <p>E1ft</p>	<p>12</p> <p>12</p>	<p>E's attempted correctly (at least 6 E's)</p> <table border="1"> <thead> <tr> <th>E_i</th> </tr> </thead> <tbody> <tr> <td>6.262</td> </tr> <tr> <td>15.369</td> </tr> <tr> <td>116.692</td> </tr> <tr> <td>42.692</td> </tr> <tr> <td>3.985</td> </tr> </tbody> </table> <table border="1"> <thead> <tr> <th>E_i</th> </tr> </thead> <tbody> <tr> <td>4.738</td> </tr> <tr> <td>11.631</td> </tr> <tr> <td>88.308</td> </tr> <tr> <td>32.308</td> </tr> <tr> <td>3.015</td> </tr> </tbody> </table> <p>Totals correct</p> <p>Attempt at combining rows Correctly</p> <p>Final column attempted (dep M1)</p> <p>2.4 to 2.5</p> <p>On their ν</p> <p>(context)</p>	E_i	6.262	15.369	116.692	42.692	3.985	E_i	4.738	11.631	88.308	32.308	3.015
	Age	Against																																																																																														
		O_i	E_i																																																																																													
	16 - 17	9	$6\frac{17}{65}$																																																																																													
	18 - 21	17	$15\frac{24}{65}$																																																																																													
	22 - 49	115	$116\frac{9}{13}$																																																																																													
	50 - 65	41	$42\frac{9}{13}$																																																																																													
	> 65	3	$3\frac{64}{65}$																																																																																													
	Total	185	185																																																																																													
	Age	Not Against																																																																																														
		O_i	E_i																																																																																													
	16 - 17	2	$4\frac{48}{65}$																																																																																													
	18 - 21	10	$11\frac{41}{65}$																																																																																													
	22 - 49	90	$88\frac{4}{13}$																																																																																													
	50 - 65	34	$32\frac{4}{13}$																																																																																													
> 65	4	$3\frac{1}{65}$																																																																																														
Total	140	140																																																																																														
O_i	E_i	$\alpha = O_i - E_i$	$\frac{\alpha^2}{E_i}$																																																																																													
26	21.63	4.369	0.8825																																																																																													
115	116.69	-1.692	0.0245																																																																																													
44	46.68	-2.677	0.1535																																																																																													
12	16.37	-4.369	1.1662																																																																																													
90	88.31	1.692	0.0324																																																																																													
38	35.32	2.677	0.2029																																																																																													
325	325		2.462																																																																																													
E_i																																																																																																
6.262																																																																																																
15.369																																																																																																
116.692																																																																																																
42.692																																																																																																
3.985																																																																																																
E_i																																																																																																
4.738																																																																																																
11.631																																																																																																
88.308																																																																																																
32.308																																																																																																
3.015																																																																																																
	Total		12																																																																																													

MS2B (cont)

Q	Solution	Marks	Total	Comments
4(a)	Sketch: 	B3	3	1 for straight line $0 \leq x \leq 1$ from (0, 0.5) to (1, 0.5) 1 for straight line $1 \leq x \leq 3$ from (1, 0.5) to (3, 0) 1 for axes [must have at least (0,0.5) (1,0) and (3,0) labelled]
(b)	$P(X \leq \eta) = F(\eta) = 0.5$ $(\Rightarrow \eta = 1 \text{ (from graph)})$	M1 A1	2	AG
(c)	$\mu = E(X) = \int_0^1 \left(\frac{x}{2}\right) dx + \int_1^3 x \left(\frac{3-x}{4}\right) dx$ $= \left[\frac{x^2}{4}\right]_0^1 + \frac{1}{4} \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_1^3$ $= \frac{1}{4} + \frac{1}{4} \left[\left(\frac{27}{2} - 9\right) - \left(\frac{3}{2} - \frac{1}{3}\right)\right]$ $= \frac{1}{4} + \frac{5}{6} \quad (0.25 + 0.83\bar{3})$ $= 1 \frac{1}{12}$	M1 A1 ml A1	4	Both integrals stated Either Correct limits used on both integrals +combined dep M1 (CAO)
(d)	Area of Δ $= P\left(X > 2\frac{1}{4}\right) = \frac{1}{2} \times \frac{3}{4} \times \frac{3 - 2\frac{1}{4}}{4}$ $= \frac{3}{32} \times \frac{3}{4} = \frac{9}{128}$ $\therefore P\left(X < 2\frac{1}{4}\right) = 1 - \frac{9}{128}$ $= \frac{119}{128} \quad (0.9296875)$	M1ft M1ft A1	3	Alternative: For $1 \leq x \leq 3$ $F(x) = 1 - \frac{1}{8}(3-x)^2$ M1ft \Downarrow $F\left(2\frac{1}{4}\right) = 1 - \frac{1}{8} \times \frac{9}{16}$ M1ft $= \frac{119}{128}$ CAO

MS2B (cont)

Q	Solution	Marks	Total	Comments												
4(d)	or			Alternative												
	$\int_{2\frac{1}{4}}^3 \frac{3-x}{4} dx \left(= \frac{9}{128} \right)$	M1 ft		$f\left(2\frac{1}{4}\right) = \frac{3}{16} = 0.1875$												
	$= 1 - \int_{2\frac{1}{4}}^3 \frac{3-x}{4} dx$	M1 ft		$P(X < 3\mu - \eta) = P\left(X < 2\frac{1}{4}\right)$												
	$= 1 - \frac{1}{4} \left[3x - \frac{x^2}{2} \right]_{2\frac{1}{4}}^3$			$= \frac{1}{2} + \boxed{\frac{1}{2} \left(\frac{3}{16} + \frac{1}{2} \right) \times 1\frac{1}{4}}$ M1ft												
	$= 1 - \frac{1}{4} \left[9 - \frac{9}{2} - \frac{27}{2} + \frac{81}{32} \right]$			$= \frac{1}{2} + \frac{55}{128} (0.4296875)$ M1ft												
	$= 1 - \frac{1}{4} \times \frac{9}{32} = \frac{119}{128}$	A1		$= \frac{119}{128} (0.930)$ A1												
	or $(1 - 0.0703125 = 0.9296875)$															
	Total		12													
5(a)(i)	$P(\text{GG or YY or RR})$ $= \frac{2}{10} \times \frac{1}{9} + \frac{3}{10} \times \frac{2}{9} + \frac{4}{10} \times \frac{3}{9}$ $= \frac{2}{9}$	M1 A1	2	(AG)												
(ii)	$P(\text{B}\bar{\text{B}} \text{ or } \bar{\text{B}}\text{B}) = \frac{1}{10} \times \frac{9}{9} + \frac{9}{10} \times \frac{1}{9}$ $= \frac{1}{5}$	M1 A1	2	(AG)												
(b)(i)	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th></th> <th>Same</th> <th>1 Blue</th> <th>Neither</th> </tr> </thead> <tbody> <tr> <th>x</th> <td>135</td> <td>145</td> <td>-45</td> </tr> <tr> <th>$P(X=x)$</th> <td>$\frac{2}{9}$</td> <td>$\frac{1}{5}$</td> <td>$\frac{26}{45}$</td> </tr> </tbody> </table>		Same	1 Blue	Neither	x	135	145	-45	$P(X=x)$	$\frac{2}{9}$	$\frac{1}{5}$	$\frac{26}{45}$	B1 B1	2	
	Same	1 Blue	Neither													
x	135	145	-45													
$P(X=x)$	$\frac{2}{9}$	$\frac{1}{5}$	$\frac{26}{45}$													
(ii)	$E(X) = 135 \times \frac{2}{9} + 145 \times \frac{1}{5} + (-45) \times \frac{26}{45}$ $= 29 + 30 - 26$ $= 33 \text{ pence}$	M1 A1	2	Multiply two rows of their table from (b)(i) AG												
(c)(i)	$E(Y) = 104 - 3E(X)$ $= 104 - 3 \times 33$ $= 5 \text{ pence}$ <p>\therefore Joanne would expect to win £5</p>	M1 A1 A1	3	OE (eg 500p)												

MS2B (cont)

Q	Solution	Marks	Total	Comments
5(c)(ii)	$E(X^2) = 9425$ $\text{Var}(X) = 9425 - 33^2 = 8336$ $\text{Var}(Y) = 9 \times \text{Var}(X)$ $= 9 \times 8336$ $= 75024$ \Rightarrow standard deviation (Y) = 274 pence	B1 B1 M1 A1	4	$(4205 + 4050 + 1170)$ $\text{sd}(X) = 91.30$ $9 \times (\text{their Var}(X) > 0)$ or $3 \times (\text{their sd}(X))$ 273.9p or £2.74
Total			15	
6(a)(i)	$\bar{x} = 43.5$ $s = 2$ ($s^2 = 4$) Assumption: Weights of boxes are normally distributed $t_{0.975} = 2.365$ 95% CI for μ : $43.5 \pm 2.365 \times \frac{2}{\sqrt{8}}$ 43.5 ± 1.6723 $\Rightarrow (41.8, 45.2)$	B1 B1 B1 B1 M1 A1	6	(AWRT)
(ii)	CI contains mean (45) Bishen's belief probably justified or [Since 45 within CI] but close to upper limit, there is some evidence that Bishen's Belief is untrue [but the evidence is not significant at 5%.] (75% of sample less than 45grams)	B1 dep B1 dep (B1)	2	Must be clear use of 45 and not 43.5
6(b)(i)	$H_0: \mu = 45$ $H_0: \mu < 45$ Test statistic: $t = \frac{43.5 - 45}{\frac{2}{\sqrt{8}}}$ $= -2.12$ $\nu = 7 \Rightarrow t_{crit} = -1.895$ \Rightarrow Reject H_0 Evidence at the 5% level of significance to support Abi's claim that mean content < 45 grams	B1 M1 A1 B1 A1 E1	6	(both)
(ii)	Type I error have/may have rejected H_0 when H_0 true or No error have/may have accepted H_0 when H_0 true	B1 B1 (B1) (B1)	2	Clear statement
Total			16	
TOTAL			75	